Improvements in Light Modulators of the Traveling-Wave Type*

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Summary—Wide-band modulation of light by means of the electro-optic effect requires a traveling-wave type of interaction, with the modulation field traveling with the same phase velocity as the light in some suitably proportioned structure. If electro-optic material is lossy at the modulation frequencies, the modulating field is strongly attenuated with a resultant low-modulation efficiency.

A scheme is analyzed here in which power is continuously fed into the light-carrying guide to make up for the attenuation as the wave progresses down the guide. By suitably tapering the coupling and the uncoupled propagation constant, the electric field can be maintained constant in the light-carrying guide and the "coupled" propagation constant in this guide can be maintained in synchronism with the light wave, thereby increasing the modulation efficiency.

Introduction

Wide-band modulation of light by means of the electro-optical effect[1,2] requires a traveling-wave kind of interaction, with the modulation field traveling with the same phase velocity as the light in some suitably proportioned structure. In Kaminow[2] an electro-optic material in the shape of a rod was centered axially in a cavity excited in an approximate TM_{013} mode. The cavity was filled with a dielectric and the dimensions were adjusted so that the microwave phase velocity approximated the light velocity in the rod.

It appears that the electro-optic material may have considerable loss at the modulation frequencies and this loss sets a limit on the useful length of action of the modulating field upon the light. In this paper, we shall analyze a scheme which should increase this useful length of interaction, resulting in a higher modulation efficiency.

Instead of using only the one waveguide containing the active material which is fed only at its input, it is proposed that one use two waveguides which are continuously coupled so that power will be fed throughout its entire length from a low-loss waveguide into the high-loss guide containing the electro-optic material. (See Fig. 1.) The phase velocity in the high-loss guide must be kept in synchronism with the light beam and this requires that the uncoupled high-loss guide propagation constant must vary with Z, the distance from the input end. To increase the efficiency, more power must be fed into the high-loss guide as Z increases so that a constant electric field will be maintained throughout the entire interaction length. To do this the coupling must also increase with Z.

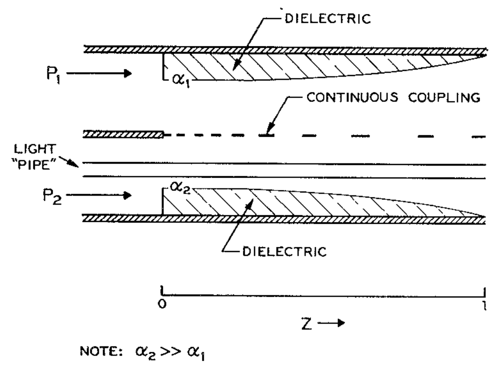

Fig. 1—Schematic of continuously coupled light-modulating (2) and feeding (1) guides. The coupling and propagation "constants" vary so that the power in guide (2) remains constant and equal to P_2 and has constant propagation constant equal to β.

It is our purpose to show how the coupling and propagation "constants" must vary in order to accomplish the above results and to compare the efficiency of this scheme with the single guide scheme in which the amplitude of the electric field continuously decreases due to the loss.

Tapered Coupled Transmission Lines with Loss

Consider two coupled transmission lines with loss. Let the input power be divided between the two guides such that

$$P = P_1 + P_2 \tag{1}$$

where P_1 is the input power to guide 1, having a loss constant α_1 and propagation constant β_1 and subscript 2 refers to guide 2 which is the light-carrying guide. Due to the large loss in the optical material

$$\alpha_2 \gg \alpha_1. \tag{2}$$

In order to insure a constant electric field, E_2, in guide 2, we require that the power in guide 2 be independent of Z, the distance from the input, *i.e.*,

$$P_2(Z) \equiv P_2. \tag{3}$$

This will require that the coupling vary with Z in a manner to be determined now.

* Received February 21, 1962; revised manuscript received, April 16, 1962.

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[2] I. P. Kaminow, "Microwave modulation of the electrooptic effect in KH_2PO_4," *Phys. Rev. Letts.*, vol. 6, pp. 528–529; May 15, 1961.

The coupled mode equations for two coupled waveguides with loss are[3,4]

$$\frac{da_1}{dZ} + [\alpha_1 + j(\beta_1 + c)]a_1 = jca_2$$

$$\frac{da_2}{dZ} + [\alpha_2 + j(\beta_2 + c)]a_2 = jca_1 \tag{4}$$

where c is the coupling coefficient and $|a_{1,2}|^2$ represent the average power in each guide.

We shall now state how $\beta_{1,2}(Z)$ and $c(Z)$ may vary in order to 1) keep $|a_2(Z)|^2 = P_2$ (constant) and 2) maintain a constant phase velocity in line 2 so that the modulating field will stay in synchronism with the light wave. We shall then verify its self-consistency.

As will become evident immediately below, the coupling must satisfy the following equation:

$$\frac{dc}{dZ} = \alpha_1 c + \frac{c^3}{\alpha_2}\cdot \tag{5}$$

This nonlinear differential equation is a Bernoulli equation[5] which is reducible to linear form by the transformation $y = c^{-2}$. The solution is then easily found to be

$$c(Z) = \frac{c_0 e^{\alpha_1 Z}}{\sqrt{1 + \kappa^2 - (\kappa e^{\alpha_1 Z})^2}} \tag{6}$$

where c_0 is the value of the coupling coefficient at $Z = 0$ and

$$\kappa^2 = \frac{c_0^2}{\alpha_1 \alpha_2}\cdot \tag{7}$$

This is plotted in Fig. 2 vs $\alpha_1 Z \equiv \zeta$ for $\kappa^2 = 0.255$.

Simultaneously, we require that the β's vary like

$$\beta_1(Z) = \beta_2(Z) = \beta - c(Z) \tag{8}$$

where β is a constant equal to the propagation constant of the light wave in the electro-optic material. Under this variation the maximum length of the coupler is determined by $c(l) = \beta$. This length will be somewhat shorter than the optimum length found below.

We shall now show that if $c(Z)$ and $\beta_{1,2}(Z)$ vary as in (6) and (8), the power in guide 2 is independent of Z and the wave propagates in guide 2 with constant propagation constant β.

If we substitute (8) into (4) the coupled waveguide equations become

$$\frac{da_1}{dZ} + (j\beta + \alpha_1)a_1 = jc(Z)a_2 \tag{9a}$$

$$\frac{da_2}{dZ} + (j\beta + \alpha_2)a_2 = jc(Z)a_1. \tag{9b}$$

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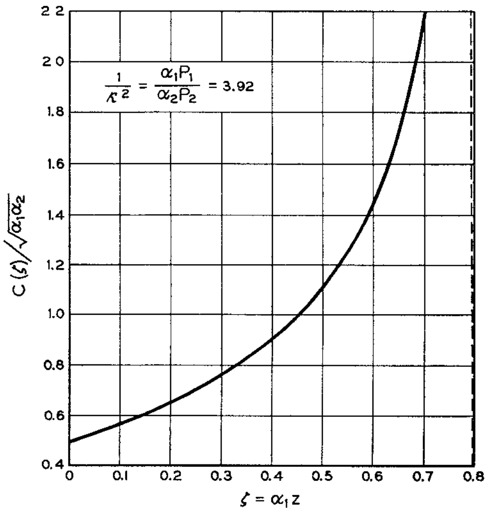

Fig. 2—Variation of coupling with distance.

We claim that $a_2(Z)$ is given by

$$a_2(Z) = \sqrt{P_2}\, e^{-j\beta Z} \tag{10}$$

where $|a_2(Z)|^2 = P_2$ is the constant power in guide 2 and β is a constant propagation constant. Now substitute (10) in (9b) and we see that

$$a_1(Z) = -j\frac{\alpha_2}{c(Z)} a_2(Z) = -j\frac{\alpha_2}{c}\sqrt{P_2}\, e^{-j\beta Z}. \tag{11}$$

Now substitute (11) in (9a) to see how $c(Z)$ must vary in order to satisfy the equation. Noting from (10) that

$$\frac{da_2}{dZ} = -j\beta a_2, \tag{12}$$

(9a) is seen to reduce to (5) and the proof is complete.

Thus, we conclude that if $c(Z)$, $\beta_{1,2}(Z)$ vary like (6) and (8), respectively, $a_2(Z)$ and $a_1(Z)$ vary as (10) and (11) as required.

Next note that c_0 is determined by (11) as follows:

$$a_1(0) = -j\sqrt{P_1(0)} = -j\frac{\alpha_2\sqrt{P_2}}{c_0},$$

or

$$c_0 = \alpha_2\sqrt{\frac{P_2}{P_1}}\cdot \tag{13}$$

Also by (7) and (13) we have

$$\kappa^2 = \frac{\alpha_2}{\alpha_1}\frac{P_2}{P_1}\cdot \tag{14}$$

Since the power in line 1 is $|a_1(Z)|^2 = P_1(Z)$, we see on using (13) and (6) in (11) that the power in line 1 varies as

$$P_1(Z) = P_1 e^{-2\alpha_1 Z}[1 + \kappa^2 - (\kappa e^{\alpha_1 Z})^2]. \tag{15}$$

The useful length of this device is presumably reached when

$$P_1(l) = P_2. \tag{16}$$

Solving (15) for l when $Z=l$ and using (14) and (16), we find that

$$2\alpha_1 l = \ln\left[1+\frac{\alpha_2 P_2}{\alpha_1 P_1}\right] + \ln\left[\frac{\alpha_1 P_1}{(\alpha_2+\alpha_1)P_2}\right]. \tag{17}$$

A quantity that measures the modulation efficiency of the device is proportional to the product of the (constant) electric field strength times the length over which the interaction takes place, *viz.*,

$$E_2 l = k\sqrt{P_2}\, l, \tag{18}$$

where the constant, k, relates the field and the power. Using (17) we have

$$E_2 l = \frac{k\sqrt{P_1}}{2\sqrt{\alpha_1\alpha_2}} \frac{2\alpha_1 l}{\sqrt{\dfrac{\alpha_1 P_1}{\alpha_2 P_2}}}$$

$$\cong \frac{k\sqrt{P_1}}{2\sqrt{\alpha_1\alpha_2}} \frac{\left\{\ln\left(\dfrac{\alpha_1 P_1}{\alpha_2 P_2}\right) + \ln\left(1+\dfrac{\alpha_2 P_2}{\alpha_1 P_1}\right)\right\}}{\sqrt{\dfrac{\alpha_1 P_1}{\alpha_2 P_2}}}$$

$$= \frac{k\sqrt{P_1}}{2\sqrt{\alpha_1\alpha_2}} \frac{\ln\left[1+\dfrac{\alpha_1 P_1}{\alpha_2 P_2}\right]}{\sqrt{\dfrac{\alpha_1 P_1}{\alpha_2 P_2}}}. \tag{19}$$

It is easy to see that $E_2 l$ will be a maximum when

$$\ln\left[1+\frac{\alpha_1 P_1}{\alpha_2 P_2}\right] = \frac{2\left(\dfrac{\alpha_1 P_1}{\alpha_2 P_2}\right)}{1+\dfrac{\alpha_1 P_1}{\alpha_2 P_2}}. \tag{20}$$

An approximate solution of this transcendental equation is

$$\frac{1}{\kappa^2} \equiv \frac{\alpha_1 P_1}{\alpha_2 P_2} \cong 3.92 \tag{21}$$

which shows how the power must be divided at the input to the guides. Since $\alpha_2 \gg \alpha_1$, it follows that $P_1 \gg P_2$.

Putting (21) into (19) we find

$$(E_2 l)_{\max} = \frac{k\sqrt{P_1}}{\sqrt{\alpha_1\alpha_2}}(0.4024). \tag{22}$$

Furthermore, we see from (17) and (21) that

$$\alpha_1 l_{\max} \cong 0.797. \tag{23}$$

From Fig. 2, it is seen that $c(l_{\max}) = \infty$ so that this is a somewhat optimistic upper limit on the interaction length.

Comparison of Tapered Coupled Guide Scheme with Single Guide Scheme

If only one waveguide is used to carry the modulating field as well as the optically active material, the electric field will vary as

$$E_2(Z) = k\sqrt{P}\, e^{(-j\beta-\alpha_2)Z} \tag{24}$$

where P is the total available power. Then this integrated action of the field over the length $l_{\max}$ is

$$\int_0^{l_{\max}} E_2(Z)dZ \cong \frac{k\sqrt{P}}{\alpha_2}\left[1-e^{-\alpha_2 l_{\max}}\right]$$

$$= \frac{k\sqrt{P}}{\alpha_2}\left[1-e^{-\alpha_2/\alpha_1 0.797}\right]. \tag{25}$$

Since $\alpha_2 \gg \alpha_1$, we have

$$\int_0^l E(Z)dZ \cong \frac{k\sqrt{P}}{\alpha_2} \cong \frac{k\sqrt{P_1}}{\alpha_2}. \tag{26}$$

Since $P=P_1+P_2$ and $P_1 \gg P_2$. Comparison of (22) and (26) shows that the scheme of coupling the power from a low-loss guide into a high-loss guide in the manner described above increases the efficiency by a factor approximately equal to

$$0.402\sqrt{\frac{\alpha_2}{\alpha_1}} \tag{27}$$

over the single high-loss guide scheme which is a considerable improvement when $\alpha_2 \gg \alpha_1$.

The principle of the scheme should be capable of being used to advantage also in situations other than traveling-wave light modulators; for example, in application to linear particle accelerators.